MERTON'S MODEL

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Merton (1974) applied Black-Scholes analysis to the problem of credit risk. He assumed that a firm has a very simple capital structure, i.e., a single liability which is due at T and requires to pay L dollars; the firm's asset follows geometric Brownian motion

$$dA_t = \mu A_t dt + \sigma A_t dZ_t$$

For bond holder, he can receive

$$\begin{cases} L & A_T \geq L \\ A_T & A_T < L \end{cases}$$

For shareholder, he can receive

$$\begin{cases} A_T - L & A_T \geq L \\ 0 & A_T < L \end{cases}$$

hence shareholders receive $\max(A_T - L, 0)$, which can be interpreted as a call option on the assets of the firm. In contrast, it is like bondholder owns the firm and sells a call to shareholder with strike L and maturity T.

If we go with Black-Scholes argument to price the equity of the firm as a function of the value of its assets, we can get

$$E_0 = A_0 \Phi(d_1) - e^{-rT} L \Phi(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{A_0}{L}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T} = \frac{\ln\left(\frac{A_0}{L}\right) + \left(r - \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}$$

Thus the equity price is calculated by taking the present value of the expected payoff under the risk neutral measure. The default occurs when the assets of the firm fall below some level.

In real world, the probability of default is given by

$$\Pr(Default) = P(A_T \leq L) = N\left(\frac{\ln\left(\frac{L}{A_0}\right) - \left(\mu - \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}\right)$$

In risk neutral world (for pricing purpose), the PD is given by

$$\mathbb{Q}[Default] = \mathbb{Q}(A_T \leq L) = N\left(\frac{\ln\left(\frac{L}{A_0}\right) - \left(r - \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}\right)$$

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It is equivalent to saying default happens if

$$Z_T < \frac{\ln\left(\frac{L}{A_0}\right) - \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$